1) Let $B=\{\vec{e}_1,\vec{e}_2\}$ be a basis of a Euclidean vector space E satisfying:

$$\begin{cases} \vec{e}_1.(3\vec{e}_1+\vec{e}_2)=2 \\ \vec{e}_2.\vec{e}_2=2 \\ \vec{e}_2.(-2\vec{e}_1+\vec{e}_2)=4 \end{cases}$$

a) Find the scalar product matrix with respect to B and the angle between the vector of the basis

b) If W is a subspace with equations (on B) $x-2y=0$, find the equations of, $W^{\perp}$ (on B)

c) Find an orthonormal basis of E.

d) Find the equations (on B) of the the orthogonal reflection with respect to W.

2) Let $B=\{\vec{e}_1,\vec{e}_2\}$ be a basis of a Euclidean vector space E satisfying:

$\|\vec{e}_2\|=\sqrt{2}$, $\vec{e}_1-\vec{e}_2$ is a unit vector, $(\vec{e}_1-\vec{e}_2)\cdot\vec{e}_2=-1$

(a) Find the scalar product matrix with respect to B and the angle that $\vec{u}=2\vec{e}_1-\vec{e}_2$ has with $\vec{e}_2$

(b) Find the equations of the orthogonal projection onto the subspace W spanned by $\vec{e}_2$ and the equations of the orthogonal reflection with respect to W.

3) Let $B=\{\vec{e}_1,\vec{e}_2\}$ be a basis for a Euclidean vector space E such that the scalar (inner) product matrix with respect to B is $\begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$,

a) Find the angle that $\vec{e}_1$ has with $\vec{e}_2$

b) If $\vec{f}$ is a linear map such that $\vec{f}(\vec{e}_1)=\frac{7}{5}\vec{e}_1+\frac{8}{5}\vec{e}_2$ and $\vec{f}(\vec{e}_2)=-\frac{4}{5}\vec{e}_1-\frac{1}{5}\vec{e}_2$, is f a symmetric tensor? If possible, find a spectral basis for $\vec{f}$ of, E

4) Consider a basis $B=\{\vec{e}_1,\vec{e}_2\}$ for a Euclidean vector plane verifying $\|\vec{e}_1\|=1$ y $\vec{e}_1\cdot\vec{e}_2=2$. If $\begin{pmatrix} \frac{2}{\sqrt{5}} & -\frac{1}{\sqrt{5}} \\ -\frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \end{pmatrix}$ is the matrix of an orthogonal tensor with respect to B, find the inner product matrix with respect to B and an orthonormal basis.

5) Let $B=\{\vec{e}_1,\vec{e}_2\}$ be a basis for a Euclidean vector space satisfying:

$$\begin{cases} \vec{e}_1.\vec{e}_2=-1 \\ (\vec{e}_2-\vec{e}_1)(\vec{e}_1+\vec{e}_2)=-3 \\ \|\vec{e}_2\|=1 \end{cases}$$

a) Find the scalar product matrix with respect to B and the angle between the vectors of the basis

b) If W is a subspace with equations on B: $x_1-x_2=0$, find the equations of the orthogonal complement of $W, W^{\perp}$ (on B)

c) If $\vec{f}$ is a linear map such that $\vec{f}(\vec{e}_1)=3\vec{e}_1+\vec{e}_2$ and $\vec{f}(\vec{e}_2)=2\vec{e}_2$, is f an orthogonal vector reflection? Justify your answer

d) Find the image under the vector rotation of angle $\frac{\pi}{2}$ of the vector $\vec{e}_2$